Calculating doubling time

Reinforcing feedback is responsible for reinforcing growth. When a performance variable grows according to a fixed fraction of its own value, the growth is exponential. When the amount of the growing action is based on previously accumulated growing actions, it is said to **compound**.

Calculating compounding growth

Let's say that you invest 1000 euros at an annual interest rate of 10%. At the end of every year, the bank pays you of 10% on the balance of your account. At the end of the first year, they pay you 1000€ x 10% = 100€ in interest. This interest is paid into your account and increases the size of your savings balance. The future value of your account is equal to the present value plus the interest earned.

We can write this as an equation: *Future value (FV) = Present value (PV) x (1+interest rate)*

At the end of the second year you earn 1100€ x 10% = 110€ in interest. Again, this amount is added to your account. As your account balance increases every year, so too does the amount of interest paid. You earn "interest on your interest". The growing action compounds according to the compounding factor.

Now the equation is: $FV_{year\ 2} = PV_{year\ 1} \times (1.1) \times (1.1)$ or $FV_{year\ 2} = PV_{year\ 1} \times (1.1)^2$ as the future value at the end of year two is simply the present value at the start of year 1 compounded twice.

This equation for compounding growth is commonly written as $\mathbf{FV = PV \times (1+r)^n}$, where FV = Future Value, PV = Present Value, r = annual interest rate and n = number of periods.

Calculating the doubling time

The doubling time is simply the number of periods (t) it takes for the PV to double. This can written as $FV = 2PV$, or $2PV = PV \times (1+r)^n$.

We need to resolve this this equation for t to calculate how long it will take for a present value to double.

(1) Divide both sides by PV to eliminate PV: $2 = (1 + r)^n$

(2) Take the natural logarithm of both sides to bring the exponent n down: $\log 2 = n \log (1 + r)$

(3) Divide both sides by log (1 + r): $\mathbf{n = \log 2/\log(1 + r)}$. **This is the exact equation for doubling time.**

Since we don't always have a scientific calculator or log tables with us, we simplify this equation and approximate the doubling time.

(1) The log of 2 is 0.7 (actually its 0.69314 but we round it off to 0.7 to simplify things)

(2) The log of 1 plus a very small number such as a percentage is approximately equal to that very small number. For example, for a growth rate of 10% (r=0.1), log(1+0.1) = 0.095 (or 9.5%); for a growth rate of 6% (r=0.6), log (1+0.06) = 0.058 (or 5.8%).

The **approximate equation for doubling time** is: $\mathbf{n = 0.7/r}$ (where r is a decimal) or $\mathbf{n = 70/R}$ (where R is a percentage).