## 3.2 Nonlinearities, butterflies and chaos

Relationships between variables in a system can be linear or nonlinear. The most common type of relationship is nonlinear, but the easiest to study mathematically and conceptually is the linear relationship.

A linear relationship is one where the output is directly proportional to the input. For example, the relationships in your savings account are most likely to be linear. For every euro you save, the bank pays the same interest rate. If you invest 100 euros at 5%, the bank will pay you 5 cents for each euro saved. If you deposit 100,000 euros into your account, the bank will still pay you 5 cents for each euro in your savings account. The output, interest earned, is directly proportional to the input, your level of savings. We can represent this relationship as a straight line. It is a linear relationship.

The output of a nonlinear system is not directly proportional to the input. For example, your local car park may offer lower rates for every extra hour you leave your car. The relationship between the number of hours you leave your car and the cost of parking is non-linear.

There are examples of nonlinear systems everywhere. What about brand recognition? When a company begins to advertise, its brand recognition increases quite quickly. However the more it spends, the less brand recognition increases per euro spent.

Motivation is another example. When you are really motivated then any motivating action by a sports coach, a manager or a teacher may have little impact. Hey, after all, you're already motivated. However at lower levels of motivation, encouragement, rewards and maybe some berating may have a greater impact. The relationship between your level of motivation and a coach's encouragement may look something like this.

Nonlinearities make systems particularly difficult to predict. Not only do we have to understand the system's structure, but also the behavior of the effect variables across different levels of cause. In fact, feedback, accumulations and nonlinear relationships make it impossible to predict the outcome of **complex nonlinear feedback systems**. One example is the economy. The economy is a complex nonlinear feedback system. The record of economists in predicting economic activity is particularly poor.

Another example of a complex nonlinear system is the weather. Have you ever noticed how weather forecasts are only accurate for a few days or the coming week? This unpredictability is commonly known as the "butterfly effect", from the work of MIT meteorologist Edward Lorenz in the 1960s. Lorenz built a mathematical model of the way air moves around in the atmosphere and noticed that weather patterns did not always behave as predicted. Small variations in the initial values of variables in his twelve-variable computer weather model would result in grossly divergent weather patterns. He discovered this when he used a rounded off value as the starting point for one of his variables, keying in 0.506 instead of 0.506127. Feedback and nonlinear relationships compounded this small difference into major differences over time. Lorenz published his findings in a now famous paper entitled "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?".

The butterfly effect doesn't mean that the butterfly was the direct cause of the tornado but rather that a small change in the initial condition of the system such as the flapping wings of a butterfly, could cause a chain of events leading to large-scale phenomena. Had the butterfly not flapped its wings, the trajectory of the system might have been vastly different. Lorenz demonstrated that the atmosphere could only be predicted two weeks ahead, the time it takes for two virtually indistinguishable weather patterns to become completely different. Even with today's more accurate satellite and computing technologies we will always have difficulties predicting the likelihood of a rainy day beyond 14 days. Why? Because the earth’s atmosphere is a complex nonlinear feedback system.

Such sensitivity to the initial starting conditions of a model is a characteristic of “chaotic” behavior. Chaotic systems are only predictable for a short while before they appear to be moving in a random fashion.

Complex and chaotic systems may be unpredictable but they are understandable. While we cannot accurately predict the precise value of a variable at a particular time, we can still study and understand the qualitative behavior of the system. By picturing systems of elements in interaction and recognizing that there is feedback, accumulations and nonlinear relationships we can better “see” the structure at work in a complex situation and better comprehend its behavior.